A NEW STATE OF MATTER

INTRODUCTION TO BOSE-EINSTEIN CONDENSATION

■ BY C. A. SACKETT AND R. G. HULET

At temperatures less than one millionth of a degree above absolute zero some types of atoms enter a quantum-mechanical state predicted by Einstein 75 years ago but realized in the laboratory only in 1995. Cass Sackett and Randy Hulet of Rice University clarify how these so-called Bose-Einstein condensates come about and how they are produced, and explain their significance.

In 1924, Albert Einstein received a paper from a young Indian physicist named Satyendra Bose in which Bose used a quantum theory to explain the spectrum of the light emitted by hot objects. Although reputedly impressed with the paper and its importance to the description of light, Einstein quickly realized that Bose's ideas could be applied to matter as well. This realization led him to predict that if one were able to cool a gas sufficiently, it would exhibit some strange and marvelous properties. This phenomenon eventually became known as Bose-Einstein condensation, or BEC. The temperatures required, however, were so low that, at the time, it seemed unlikely BEC could ever be seen. Although the theory Einstein developed was later extended and applied to a variety of other important systems, BEC in the basic form he first imagined did in fact prove elusive.

It was not until 1995 that atomic physicists were finally able to observe BEC in the laboratory. Due to the significance of this achievement, the years of effort that it had required and its association with the popular name of Einstein, the results have generated quite a bit of publicity— or, in any event, more than a typical physics experiment. Research in the field exploded, and there are now dozens of groups investigating the properties of BEC, with important new results being obtained almost monthly. Because of this excitement, there has been substantial interest in trying to explain to those outside the physics community just what exactly BEC is.

However, BEC is an explicitly quantum phenomenon, and quantum mechanics is a strange and counterintuitive theory. This is part of what makes BEC so interesting, as it offers a chance to see the world behaving in an unexpected and unusual way. But it is a double-edged sword: the very thing that makes BEC interesting also makes it challenging to explain in ordinary terms. Nonetheless, we believe meeting this challenge to be worthwhile. Quantum mechanics has given physicists new ways to think about the basic nature of the world around us, and the better these ideas can be shared, the more fruitful they are likely to be.

PHASE TRANSITIONS

So what is BEC? To start with, it is a phase transition, which is the name for a process in which a macroscopic collection of particles changes physical state. BEC occurs when certain types of particles are cooled to sufficiently low temperatures. The criterion for the transition to occur is that the average distance between particles must be smaller than their quantum "deBroglie wavelength." The meaning of these statements will be discussed further later in this article, but the first point to note is that the temperatures required are low indeed. DeBroglie waves are a reflection of the wave/particle duality principle of quantum mechanics, which states that a particle moving with momentum p must in some respects be considered as a wave with a wavelength of h/p, where h is Planck's constant. Planck's constant is very small (6.6×10^{-34} Joule), so if several particles are to be found within a wavelength, either the particles must be packed together closely or else p must be small enough to make the wavelength large. Small momenta are only obtained by cooling the particles down until they move very slowly, so the highest transition temperatures will be found in the densest materials.

In fact, BEC is related to the phenomena of superconductivity in metals and superfluidity in liquid helium. Both of these take place in condensed matter with densities on the order of 10^{23} atoms/cm^3, and at temperatures of a few degrees Kelvin—already very cold compared to room temperature of 300 K. At these densities, however, atoms in the material strongly interact with each other, and the interactions greatly affect the system's behavior. To realize the pure physics envisioned by Einstein, the conditions for BEC must be achieved in a low-density gas, where interactions are almost negligible. This requires temperatures yet a million times colder. Some of the experimental techniques used to reach these temperatures will be discussed later.

We have defined BEC as a phase transition; the gas is cooled in order to reach the transition point. But what does this mean? In general, as we have said, in

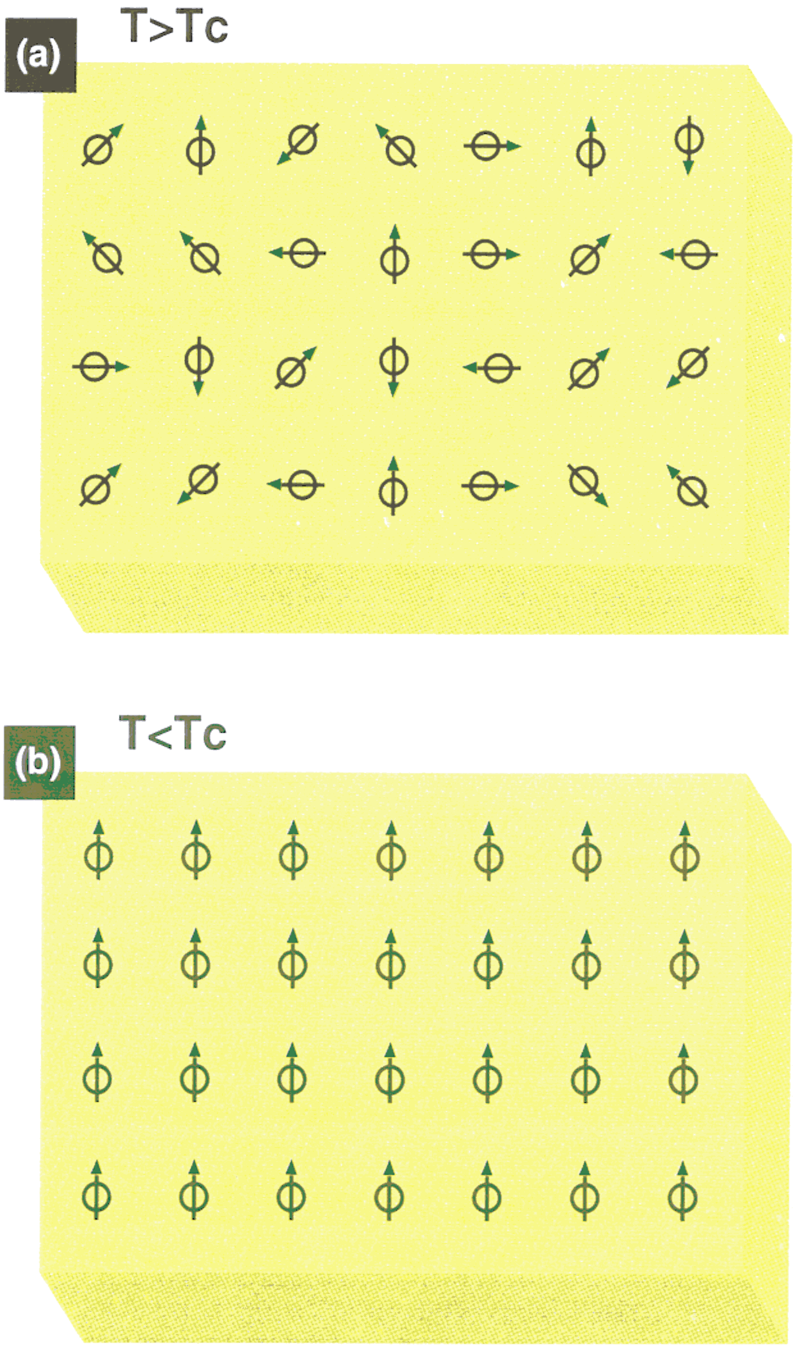

Figure 1: Illustration of the Curie phase transition in a magnet. Magnetization arises from the collective action of many individual atomic moments. (a) At high temperatures, thermal motion causes each atom to point in a random direction, so no overall magnetic field is present. (b) As the crystal is cooled, the atomic fluctuations decrease until a point is suddenly reached when the moments snap into place. Below this temperature, the crystal acts as a magnet. This is analogous to BEC, but during BEC atoms develop a uniform "quantum phase" instead of a net magnetization.

When Bose-Einstein condensation occurs in a gas of atoms, the atoms develop a uniform quantum phase in much the same way that the atoms in a magnet develop a uniform orientation.

a phase transition a macroscopic collection of particles changes physical state. An apt example from daily life is the condensation of water from the air as dew. In this case, the two phases are readily distinguished: water droplets are dense and almost incompressible, and have a well-defined boundary separating them from the surrounding vapor. A Bose-Einstein condensate, however, can generally not be so easily identified by simple observation. A better way to think of the change of state occurring in BEC is by analogy with the Curie transition in a magnetic material. When a magnet is heated sufficiently, it becomes demagnetized as the atomic magnetic moments composing it are randomized by thermal motion. If the material is then gradually allowed to cool, the magnetization abruptly reappears at a critical temperature known as the Curie point. The transition is abrupt because, once a few neighboring atoms find themselves aligned, they tend to force other, nearby atoms to align as well, spreading the order rapidly through the material. Figure 1 illustrates this process.

The Curie transition is easy to measure with a magnetometer, but imagine trying to detect it without such a device. When a hot piece of iron is cooled below its Curie temperature of 1043 K, it does not expand, contract, change color or exhibit any other obvious signature. The transition could only be detected by monitoring subtle changes in the heat capacity of the material, or by probing the atomic alignment directly by, for example, neutron scattering. Nonetheless, it marks a fundamental and significant change in the material, as atoms from one side of the magnet to the other all line up. When Bose-Einstein condensation occurs in a gas of atoms, the atoms develop a uniform quantum phase in much the same way that the atoms in a magnet develop a uniform orientation. Unlike a magnetic moment, this phase can never be observed by direct means, but it has a variety of effects which have been seen.

QUANTUM WAVES

This "quantum phase" is in fact another consequence of the wave-like nature of atoms, and it can best be understood by analogy with other wave-like processes. In particular, a good comparison can be made with light. Light has long been known to consist of electromagnetic (EM) waves, and entire technologies have grown up around exploiting its wave nature. When light is considered as a wave, the notion that it has a definite phase is easy to comprehend: at each point in space, the EM field oscillates between a minimum and maximum value in a regular way. The phase of the wave then refers to the timing of the oscillations. (Note that the word "phase" here is used with a meaning different from that in "phase transition.") Of course, this begs the question of what exactly an electromagnetic field is, but most of us have experienced socks clinging together in our laundry due to electric fields, and magnets sticking to the refrigerator due to magnetic fields. When these fields combine and oscillate quickly enough, what we perceive is light.

Waves that oscillate in a regular way are said to be coherent. Although it makes sense to think about a phase in this context, light from ordinary sources, such as an incandescent lamp, doesn't have a phase after all. Light from a lamp is emitted by individual atoms in the bulb, but these atoms act independently and have random phases with respect to one another. So, although an EM field is present, it is randomly fluctuating rather than smoothly oscillating, and a value for the phase cannot be defined. We call this kind of light incoherent. In sound waves, we would associate a well-defined phase with a definite musical tone, while incoherent sound would just be noise; Figure 2 illustrates the difference. Although most natural sources of light are incoherent, coherent light waves do exist in the form of lasers. In a laser beam, the field oscillates smoothly and uniformly, and it is this fact that makes laser beams useful both in physics and in everyday applications, like supermarket scanners. The difference between a normal gas of atoms and a Bose condensate is analogous to that between ordinary light and laser light. In a normal gas, there is an "atom field" present, but it fluctuates randomly, while in a condensate the field oscillates smoothly.

The example of light also helps explain the relation between the wave and particle aspects of a Bose condensate, because just as atoms must in some respects be thought of as waves, a beam of light must in some ways be considered as a stream of particles. We call these particles photons, and they can be detected directly with a sensitive enough instrument. Each photon is associated with a specific "mode" of the EM field, where a mode is just a particular configuration of waves in space and time. Using the example of another type of

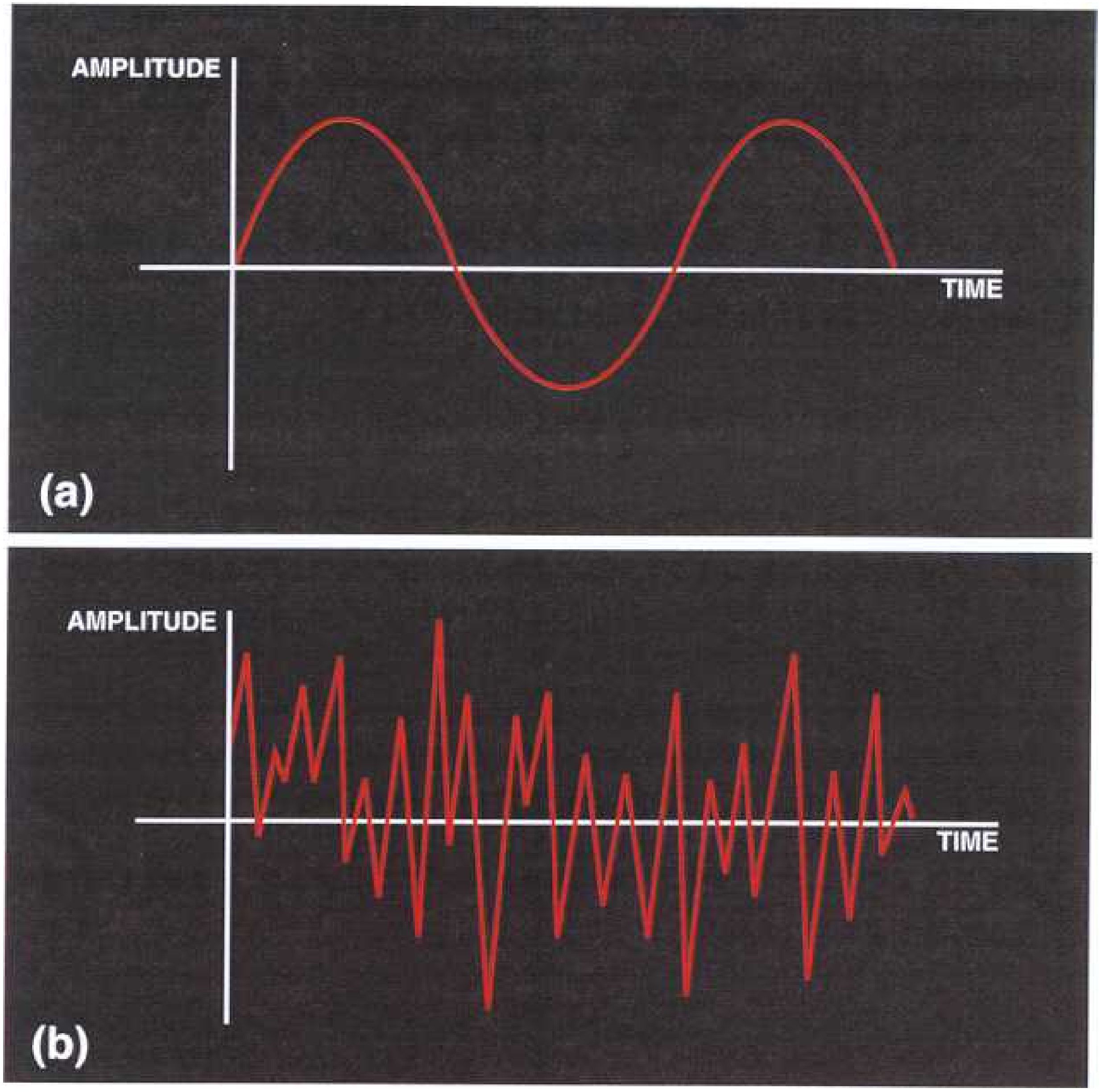

Figure 2: Existence of a phase. Both plots show a signal oscillating in time. In (a) the oscillation is smooth and uniform so that a phase for the oscillation can be defined, while in (b) the signal fluctuates randomly, with no definite phase. The signal in (b) might represent the sound waves of static noise, the light waves emitted by an incandescent bulb, or by the atom waves in an ordinary gas. The signal in (a) might represent the sound of a pure musical tone, light coming from a laser, or the atom wave associated with a Bose condensate.

The deBroglie wavelength roughly specifies the spatial extent of a field mode, so if two atoms are within a wavelength of each other, they must be in the same mode. And once this happens, it suddenly becomes likely for many of the atoms to be in that mode.

wave—sound waves—we can take a pure tone emanating from a given source as a mode. Just as any sound can be constructed from the right combination of pure tones, any kind of light can be thought of as a combination of EM field modes. The number of photons associated with a mode, then, describes the amplitude with which the mode is oscillating; the more photons there are, the more energy is in the mode.

Ordinary incoherent light has much less than one photon per mode, but many modes are excited. In wave terms, there is a lot of energy in the wave as a whole, but very little in any particular mode. That is why the field appears noisy: no particular configuration of the field is oscillating more than any other. Similarly, in a normal gas the probability is low for any particular mode of the atom field to be occupied. Because each mode oscillates independently from the rest, the different modes tend to cancel and wash out the wave effects entirely. Because of this, the gas can be described reasonably well as a collection of simple particles, which is how we usually think of it.

A laser, in contrast, characteristically has many photons in a single EM mode, which makes wave effects very important. The same is true of a Bose condensate. Although there are important differences between the two phenomena, the wave-like behavior in both is quite similar. It has been observed that a Bose condensate forms a standing wave when spatially confined, that a condensate expanding from a small source exhibits diffraction, and that two condensates can interfere both in space and in time when overlapped (see Figure 3). All these effects are very familiar to laser physicists, as well as to those working in other fields dealing with waves, like acoustics and seismography.

WHY BEC OCCURS

Although laser light and Bose condensates are in many respects analogous, the two phenomena come about in very different ways. The production of laser light relies on the ease of creating photons from other forms of energy. In a typical laser, a collection of energetically excited molecules, called the "gain medium," is enclosed by a pair of mirrors. As the molecules relax to their ground state they emit photons, a fraction of which are reflected by the mirrors back to the gain medium. The electric field of this reflected light induces other molecules to radiate their energy into the same mode, thus amplifying the reflected light. As this continues, the amplitude of radiation in the mode builds up and lasing occurs.

Making a BEC in the same way would be difficult, since transforming other forms of energy into matter is impractical. Atoms cannot be simply emitted into a field mode, but must be transferred to it from elsewhere. Also, BEC is a thermodynamical process, and describes the equilibrium state of the gas. In contrast, a laser is in an explicitly non-equilibrium state, and is only maintained by continual addition of energy to the system through replenishment of the gain medium. So, although the analogy of a Bose condensate to a laser beam does help explain the properties of BEC, it doesn't answer the question of why the condensate forms.

Surprisingly, the analogy to the Curie transition also fails to address this issue. The Curie transition, like all phase transitions except BEC, occurs because of interactions between the constituent particles. In the case of a magnet, neighboring atoms are coupled by electronic forces and, in the right circumstances, this coupling creates a tendency for the atoms to align. BEC, in contrast, is possible even in a perfectly ideal non-interacting gas and is observed in very nearly ideal gases.

The answer lies in statistics. A Bose condensate forms for no other reason than that it is the most probable configuration of a sufficiently cold collection of atoms. The standard calculation is worked out by every advanced physics student in college, but a simple example serves better to illuminate the underlying idea and requires only a basic understanding of probabilities. Rather than considering atoms, which can be in an infinite number of different states, we can make the point by considering a finite system. In particular, imagine a set of N "quantum coins," identical particles which can each be in either of two states, heads "H" or tails "T", with equal probability. What is the probability P that all of the coins are in state H? If the coins were ordinary, distinguishable objects, each possible state of the collection as a whole could be labeled by an enumeration of the states of each coin. Since each coin has two possibilities, there are 2^N distinct enumerations. The probability of any particular configuration, such as "HHH....H," would therefore be 2^{-N}.

If the coins are truly identical, like all atoms of the same element, then it is fundamentally impossible to say whether a particular coin is in a particular state. The configurations "coin 1 in H and coin 2 in T" cannot be counted as distinct from "coin 1 in T and coin 2 in H." All that can be said is that there is one coin in H and one in T, so that only one configuration can be counted. Each distinct configuration of N coins can thus be labeled simply by the number of coins in state H, and each such configuration will be equally likely. Since the occupation number can range from 0 to N, the probability P of any particular configuration (say, k coins in H and $(N-k)$ coins in T) is $1/(N+1)$. For even moderately large N, this is tremendously greater than 2^{-N}; with 100 coins for instance, $1/(100+1)$ is roughly 10^{-2}, while 2^{-100} is about 10^{-30}.

As it turns out, many real particles do behave in just this way. Those that do are termed bosons. Photons and hydrogen atoms are bosons, among others. But other particles, such as electrons and protons, behave differently. These are called fermions (named after Enrico Fermi, who developed the statistics governing their behavior),

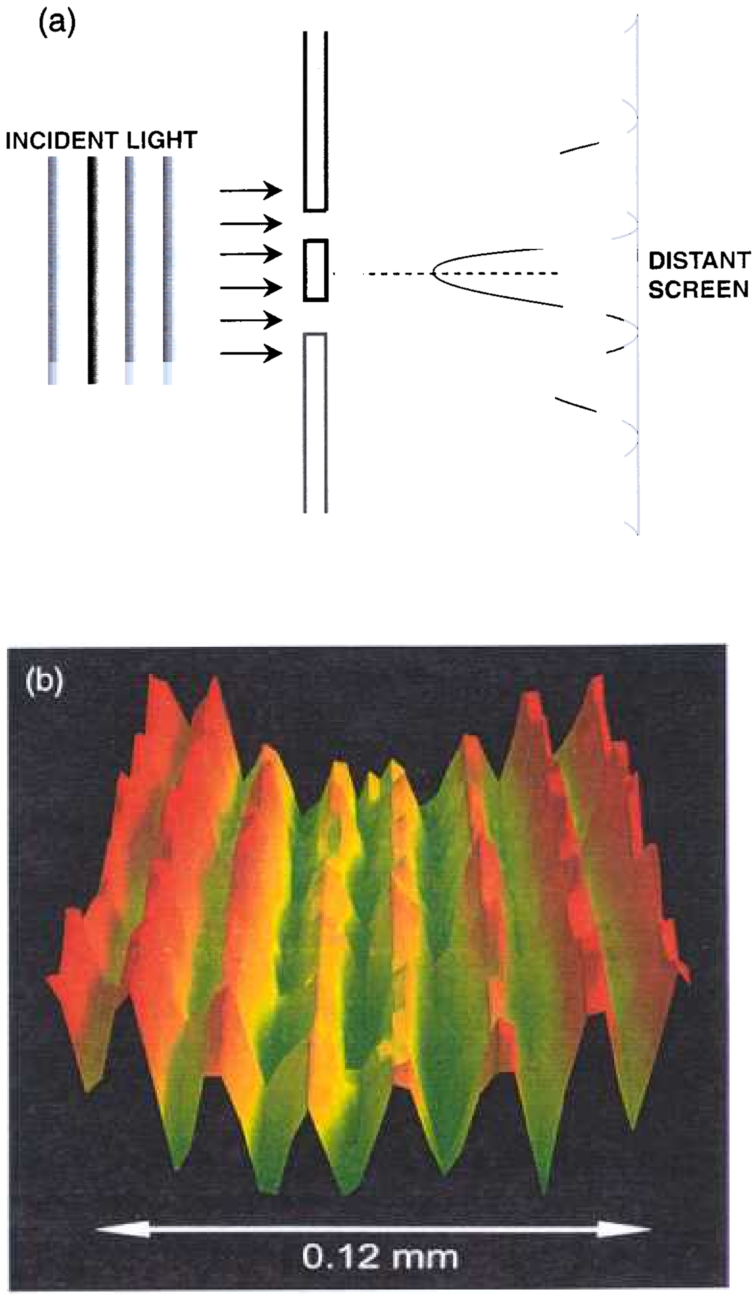

Figure 3: Wavelike behavior of a Bose condensate. A classic experiment in optics is Young's double-slit experiment, illustrated in (a). Here a laser or other coherent light source is used to illuminate two small slits in a screen, and the transmitted intensity is observed some distance away. Because light is a wave, it passes through both slits together, in essence forming two waves on the other side of the screen. These two waves combine and interfere, generating the characteristic oscillating pattern shown. A similar experiment was performed by Wolfgang Ketterle and his group at MIT using a Bose condensate instead of light. His results are shown in (b), and exhibit a similar pattern of interference fringes. (Reprinted with permission from the cover of the 31 January 1997 issue of *Science*. © 1997 American Association for the Advancement of Science.)

In order to see how statistics can bring about an abrupt phase transition, we need to allow a variable probability for observing our coins to be in state H. In a physical system, this occurs because the probability of finding an atom in a state with a given energy depends on the temperature. The higher the temperature, the more likely it is that a high-energy state will be occupied. So, if we imagine that state H has higher energy than state T, then the probability *q* of a single coin being in H is analogous to the temperature of a real gas. So how does the probability for all *N* coins to be in H depend on *q*? Our first guess might be just q^N, and this is the right relative probability, in the sense that the chance of finding *k* coins in H and therefore *N-k* coins in T is proportional to $q^k(1-q)^{N-k}$. However, the constant of proportionality must be chosen to ensure that the sum of all the probabilities is 1. So, we really have $P = Aq^N$, with

$$A\,[q^N + q^{N-1}(1-q) + q^{N-2}(1-q)^2 + \cdots + q(1-q)^{N-1} + (1-q)^N] = 1$$

We could easily program a computer to solve for *A* directly, but it is a little more interesting to work out the math. (People who really don't like math can just skip this paragraph and look at the answer in Figure 4.) If we divide through by q^N, we get

$$A\,\{1 + (1-q)/q + [(1-q)/q]^2 + \cdots + [(1-q)/q]^{N-1} + [(1-q)/q]^N\} = q^{-N}.$$

The expression in the brackets is an example of the famous geometric series, which can be summed by a simple trick. If we define $G = 1 + x + \cdots + x^N$, then $xG = x + x^2 + \cdots + x^{N+1}$. When these two are subtracted, most of the terms cancel, leaving $G - xG = 1 - x^{N+1}$. This is easily solved for *G*, to give

$$G = (1 - x^{N+1})/(1 - x).$$

Replacing *x* by $(1-q)/q$ and solving our equation for *A*, we get

$$A = (2q - 1)/[q^{N+1} - (1-q)^{N+1}],$$

which is all we need to evaluate our total probability *P*. The result is plotted in Figure 4. We indeed see a discontinuous jump at $q = 1/2$, indicating a phase transition of some kind.

Figure 4 shows that whenever one would expect the majority of the coins to be in H, there is a reasonably large probability for *all* of them to be in H. Although the detailed calculation of BEC in a gas is a bit more complicated, it follows the same logic and gives a similar final result.

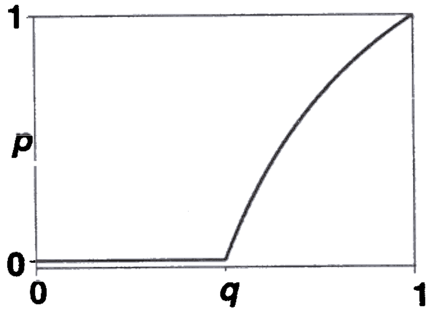

Figure 4: Calculated probability *P* for *N*=1000 quantum coins to all be found in the state H, as a function of the probability *q* of finding a single isolated coin to be in H. For small *q*, *P* is nearly zero, but above $q = 1/2$, *P* abruptly starts to increase. This discontinuous change is characteristic of a phase transition, and arises in the same way as BEC.

and have the property that no two identical fermions can ever be in the same state. In wave terms, no mode of a "fermion field" can have more then one unit of excitation. As might be guessed, BEC only occurs for bosons, since it requires a single atom field mode to be highly occupied.

Our argument with the coins demonstrates that a set of identical particles is far more likely to be found all in the same state than a similar set of distinguishable particles. This is basically what causes BEC; the sidebar shows in more detail how a phase transition comes about. Similar reasoning for a real gas shows that whenever one expects to find more than one atom in a field mode, it is likely that a reasonably large fraction of the atoms will be in that mode. This then ties back to our original definition, that BEC occurs when a gas of atoms is cold and dense enough for the atoms' deBroglie waves to overlap. The deBroglie wavelength roughly specifies the spatial extent of a field mode, so if two atoms are within a wavelength of each other, they must be in the same mode. And once this happens, it suddenly becomes likely for many of the atoms to be in that mode. This is BEC.

MAKING BEC IN THE LAB

Of course, understanding the theoretical reasoning behind Einstein's prediction is not the same thing as actually bringing about the required conditions in the laboratory. (In point of fact, the difference is about 70 years of progress in physics.) So how is BEC achieved experimentally? There are three main ingredients.

The first task is to hold onto the atoms while they are being cooled. An ordinary container will not work, because it is impossible to get bulk materials cold enough to allow BEC in a gas. This problem was solved using magnetic traps, which make use of the fact that many atoms have relatively strong magnetic moments. An atom with a magnetic moment will interact with magnetic fields just like any other magnet, so ordinary sources such as electromagnets or permanent magnets can be used to exert a force on the atoms. The right combination of sources can yield a trap, in which an atom is pushed back towards the center of the trap no matter which way it tries to move. This way, the atoms are confined without any physical contact with the room-temperature apparatus. Figure 5 depicts the magnetic trap at Rice University, which employs six permanent magnets.

Since BEC is easier to achieve at higher densities, these traps are usually made as strong as possible in order to generate tight confinement. Nonetheless, the traps are rather shallow, and they can only hold onto atoms that are moving relatively slowly. The second ingredient to BEC, then, is a way to cool the gas down enough to load the magnetic trap. This was originally accomplished using laser cooling. This technique can be used to slow atoms down to temperatures of 1 mK or lower and has seen application in a huge variety of experiments. Its importance was recognized in the awarding of the 1997 Nobel prize to Steven Chu, Claude Cohen-Tannoudji and William Phillips for their contributions to its development. A good description of the process can be found in "Atomic Trampolines" by E.A. Hinds and I. Hughes in *Science Spectra* Issue 16,

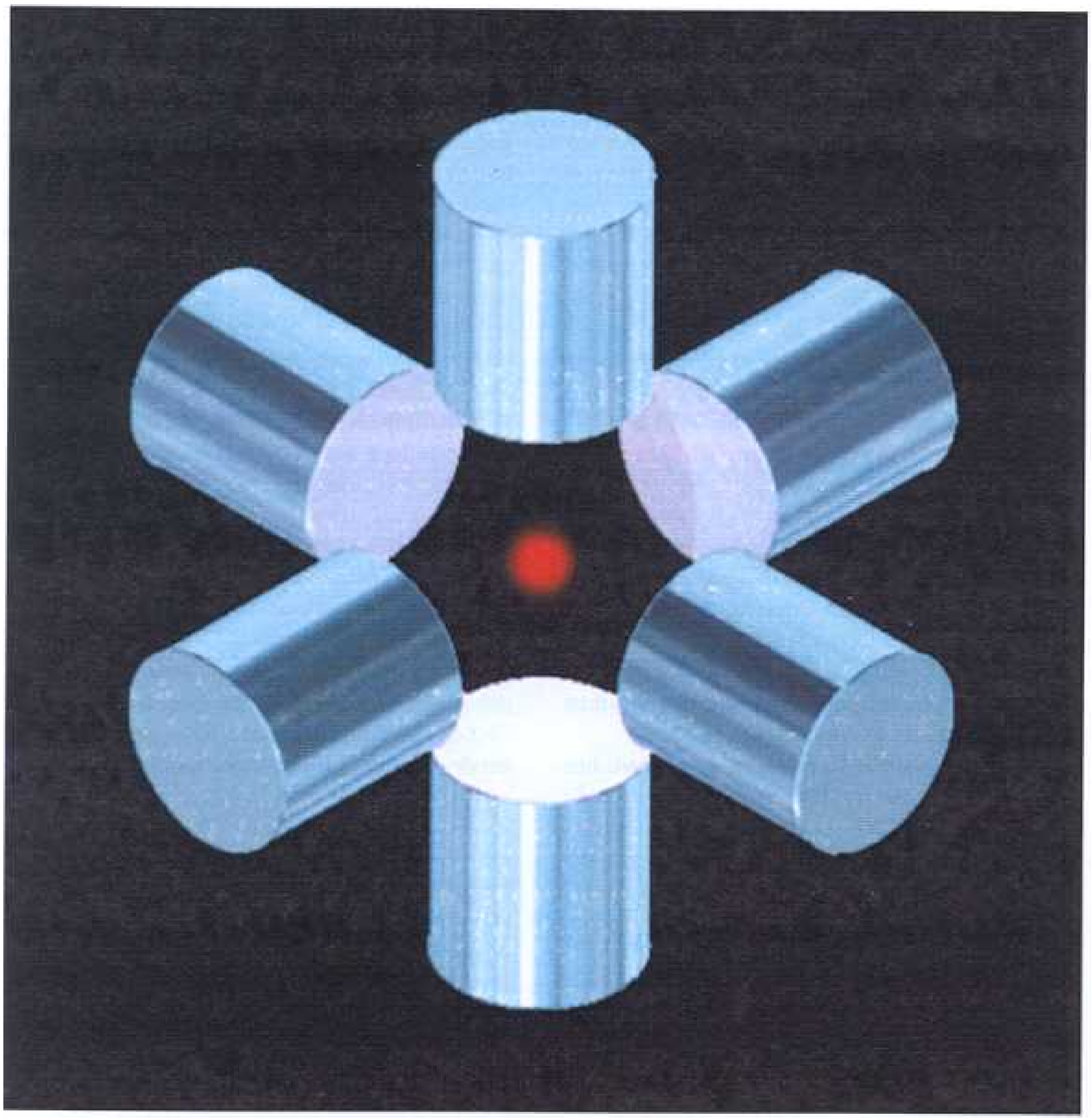

Figure 5: Drawing of a magnetic trap used at Rice University. The six cylinders are Nd-Fe-B rare-earth magnets, each about 8 mm in diameter, aligned to produce a local magnetic field minimum at the center of the structure.

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p. 66 (1999), but for the purposes of BEC, it is really only necessary to understand that by firing suitably configured laser beams into a gas, the gas can be cooled enough to load a trap.

In parallel with the development of laser cooling, other groups demonstrated that magnetic traps could also be loaded by simply cooling the gas down with conventional cryogenic techniques. This method has led to the successful observation of BEC in atomic hydrogen. Because of several technical difficulties involved in having a cryogenic apparatus, laser cooling appears to be a more popular approach. However, laser cooling can only be applied to a fairly limited group of atoms, while the cryogenic method is more general.

Unfortunately, neither laser cooling nor conventional cryogenics have been able to reach low enough temperatures and high enough gas densities for BEC. For instance, in the experiment at Rice, we load approximately 10^9 lithium atoms at a temperature of 250μK and a density of $3x10^{11}$ cm^{-3}. Under these conditions, the mean number of atoms in the volume of a cubic deBroglie wavelength is only about $7x10^{-5}$, much less than one. So, another method must be used to cool the gas further.

Several possibilities have been proposed, but the one that all successful BEC experiments to date have used is evaporative cooling. This simple idea relies on the fact that atoms in a thermal gas have a broad distribution of energies. If the highest-energy portion of this distribution is removed, then the average energy of the gas can be significantly reduced, while the total number of atoms is only slightly decreased. Furthermore, collisions between the atoms will continually replenish the high-energy tail of the distribution, so the evaporative cooling procedure can be applied over and over. In typical experiments, around 99 percent of the atoms end up being removed in the cooling process, but the reduction in temperature, of up to a factor of 1000, is more than enough to make up for the loss in density.

These techniques were combined to achieve BEC in 1995, first with rubidium atoms and shortly thereafter with lithium and sodium. More recently, BEC was also observed in hydrogen. At this point, the basic existence of the quantum phase has been confirmed in several ways—wavelike interference effects have been observed both in space and in time, and the characteristics of various "atom field" modes, such as coherence lengths, have been explored.

Current experiments are now focusing on the effects of interactions between the atoms of the gas. Although weak, these interactions have a surprisingly large impact on the nature of BEC, and give rise to a very rich set of phenomena. As just one example, our experiments at Rice deal with the isotope ^{7}Li. It turns out that, at low temperatures, the interaction between ^{7}Li atoms is attractive. For several decades, it was believed that attractive interactions such as these would prohibit the formation of BEC entirely. The problem is that the atoms in the condensate are moving too slowly to push against anything with much force, so that there is nothing to resist the attractive force of the interactions. The condensate will then simply collapse upon itself, undergoing a sort of microscopic implosion. The collapse only stops when the density becomes so high that atoms begin to combine into molecules, a process that releases so much energy that the gas is blown out of the trap. Instead of a condensate, you just get a bunch of hot molecules.

Although this argument was carefully worked out by theorists starting in the 1950s, and revisited as recently as 1994, BEC in ^{7}Li was nonetheless observed. An important aspect of the eventual experimental realization was neglected in the theories: the condensates that we make are very small in size, typically several micrometers in diameter. If you wish to localize an atom in a small condensate, then according to Heisenberg's famous uncertainty principle, there must be a corresponding uncertainty in the atom's momentum. A localized atom, therefore, can never have truly zero energy and the more tightly the atom is confined, the higher its so-called zero-point energy must be. The result is a small "quantum pressure" that enables the condensate to resist the collapse as long as the interaction forces are not too strong. Since the strength of the interactions increases with the number of atoms in the condensate, the quantum pressure is sufficient to stabilize the condensate as long as the number of atoms in it is not too large. For ^{7}Li, this limit is about 1200 atoms, a value we have experimentally verified. When this limit is exceeded, the condensate does indeed seem to collapse, which is itself an interesting process that we have begun to study experimentally. Continued exploration in this and similar areas will undoubtedly help boost our understanding of these interacting many-body quantum systems.

Just as lasers have found applications far outside their original domain of quantum optics, most researchers are optimistic that Bose condensates will be useful in a variety of different applications and physics experiments. For instance, "atom lasers" made by coupling atoms out of a condensate might be used as surface probes, or as sources for atom interferometers that could measure gravity or rotation far more accurately than the best present-day sensors. We believe, however, that even aside from its potential utility, BEC represents a unique pedagogical resource, in that it provides a clear physical demonstration of some of the strange predictions of quantum mechanics. We hope that the description given here will help this resource to live up to its potential. S

SUGGESTED READING

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