Measurement of a Weak Value *

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"Weak measurements" are measurements in which the coupling between the measuring device and the observable to be measured is so weak that the eigenvalues of the observable are not resolved. Under certain circumstances the corresponding eigenfunctions can be made to interfere, producing a measurement result which is outside the allowed range of the observable's eigenvalues. We present the first measurement of this so-called "weak value" using an optical experiment. In our experiment, the small displacement between the two orthogonally polarized components of a laser beam passed through a birefringence crystal is measured. We use a numerical simulation to show that this phenomenon may be practical for detecting and amplifying small effects.

1. Introduction

A measurement in quantum mechanics generally consists of two elements, a measuring device and the system to be measured. In an ideal measurement, the interaction between these two elements results in the measurement of an eigenvalue of an observable of the system and the system is left in the corresponding eigenstate. In a nonideal measurement, the interaction strength between the measuring device and the observable to be measured is insufficient to resolve the eigenvalues of the observable. Therefore, the measurement does not leave the system in one eigenstate of the observable. Recent theoretical and experimental work has examined this class of measurements which have been called "weak measurements" [1]. Weak measurements are of fundamental interest because quantum mechanical measurements are never ideal. The strength of any measurement can be characterized on a continuous scale which extends from weak to ideal. Since all experimental measurements fall somewhere between these extremes, an understanding of nonideal measurements is important in understanding the quantum mechanical measurement process.

In a recent paper Aharanov, Albert, and Vaidman (AAV) introduced the concept of a "weak value" of an observable [1]. To measure the weak value, systems are initially pre-selected via a nearly ideal measurement in some eigenstate $|\Psi_{in}\rangle$ of an observable. The systems then interact with a measuring device which is weakly coupled to the observable A. Finally the systems are post-selected via another nearly ideal measurement in some eigenstate $|\psi_f\rangle$ of some other observable. In this process, the "weak value" of an observable with operator $\hat{\mathbf{A}}$ is defined by AAV to equal

$$A_w = \frac{\langle \Psi_f \,|\, \hat{\mathbf{A}} \,|\, \Psi_{in} \rangle}{\langle \Psi_f \,|\, \Psi_{in} \rangle}, \tag{1}$$

where $\hat{\mathbf{A}}$ is the operator corresponding to observable A. A_w can be the result of a measurement of A, and furthermore can be much larger than any of the eigenvalues of $\hat{\mathbf{A}}$ when $|\Psi_{in}\rangle$ is nearly orthogonal to $|\Psi_f\rangle$. This strange result is due to interference between the unresolved eigenstates of $\hat{\mathbf{A}}$ which are projected onto the post-selected basis. The theory of this effect has been described in several recent publications [1–4].

2. Experiment

Duck, Stevenson, and Sudarshan proposed an optical experiment to demonstrate the measurement of a weak value [2]. In this experiment, a laser beam passes through a birefringent plate which displaces the two orthogonal linear polarizations of the beam by an amount much smaller than the Gaussian waist of the laser beam w_0. In this sense, the measurement of the polarization of the light is weak. Our realization of this experiment is depicted schematically in Fig. 1 and described in more detail in [5]. The beam from a frequency stabilized He-Ne laser was collimated and focused through the following optical elements. Polarizers performed the pre- and post-selection. The pre-

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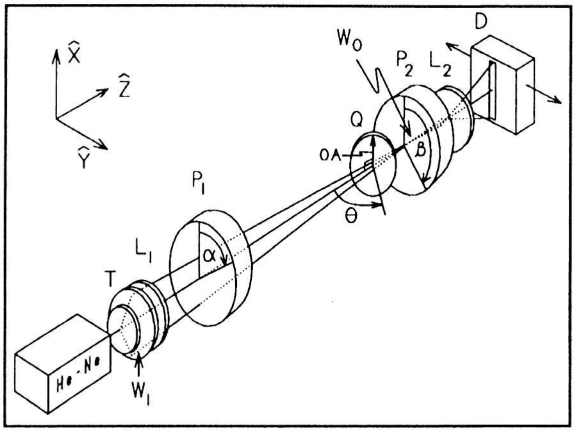

Fig. 1. Schematic of optical experiment.

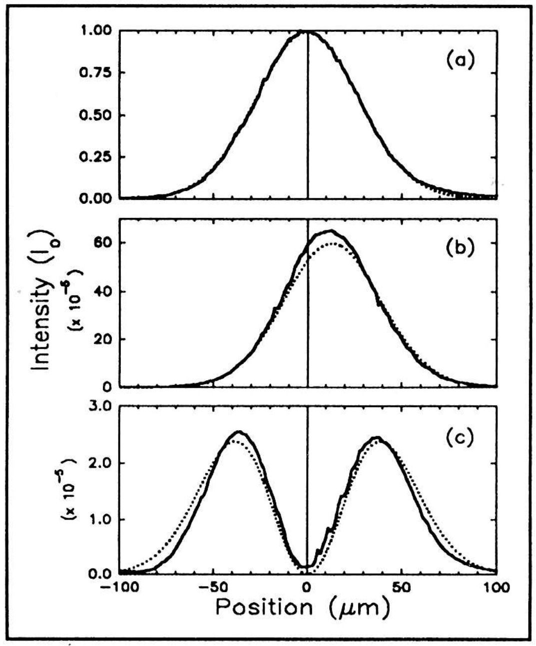

Fig. 2. Solid lines: data; dotted: fits. (a) $\langle \Psi_{in} | \Psi_f \rangle = 1$; (b) $\langle \Psi_{in} | \Psi_f \rangle \ll 1$; (c) $\langle \Psi_{in} | \Psi_f \rangle = 0$.

selecting polarizer was oriented at $\alpha = \pi/4$ with respect to the x-axis, and the post-selecting polarizer was oriented at an angle β with respect to the x-axis. The weak measurement was performd by passing the laser beam through a birefringent quartz plate oriented with the optic (extraordinary) axis along the x-axis. The plane of the plate was rotated about the optic axis by an angle θ with respect to the propagation (z) axis. This plate produced a small relative shift in the y-direction between the two orthogonal polarization components of the laser beam. At the location of the second polarizer, $w_0 = 55$ µm. Immediately after the polarizer the light was projected by a short focal length lens onto a photodiode, which was scanned across the distribution. The birefringence induced displacement a calculated from the measured indices of refraction and the known thickness of the plate is $a = 0.64$ µm.

Figure 2 shows the results of the experiment. The solid lines are data and the dotted lines are fits to the data, for which a is the fitted parameter. Figure 2a corresponds to $\alpha = \beta = \pi/4$, so that $| \Psi_{in} \rangle = | \Psi_f \rangle$. The detected intensity distribution consists of two slightly displaced and unresolved Gaussians. Figure 2b corresponds to $\alpha = \pi/4$ and $\beta = 3\pi/4 + 0.022$. The centroid of the distribution is shifted by $A_w = 12$ µm or almost 20 times a. For the data shown in Fig. 2c, $\alpha = \pi/4$ and $\beta = 3\pi/4$, corresponding to crossed polarizers, or orthogonal initial and final states. The resulting signal is due to destructive interference between the two shifted Gaussians. However, A_w is undefined for this case. Physically, the weak measurement can no longer be considered weak, since it introduces a non-negligible coupling between the initial and final states. The fits to the data of Figs. 2b and 2c give $a = 0.65 \pm 0.05$ µm and 0.62 ± 0.04 µm, respectively.

3. Application to Measurements of Small Effects

Is this phenomenon potentially useful for measuring small effects? In particular, we would like to know if the interference between eigenstates corresponding to unresolved eigenvalues leads to a distribution in which a small parameter (a in the experiment above) can be more accurately extracted than from the uninterfered distribution. The answer to this question is not obvious *a priori* since the interference causes a loss in signal intensity, in addition to the strong modulation in signal shape.

In order to answer this question, we numerically generated a number of distributions corresponding to various values of a, signal intensity I, and to various amounts of overlap between the pre- and post-selected states (i.e. various α and β). Random noise was then added to these distributions. We chose to add only statistical noise representing random fluctuations in the detected signal intensity, rather than noise attributed to systematic effects. Therefore, the numerically generated data correspond to ideal "shot-noise" limited "experiments". The relative noise is proportional to $I^{-1/2}$, so that the uninterfered data are relatively less noisy than the data corresponding to destructively interfered distributions.

A fitted value for a was obtained for each parameter combination. The deviation of the fitted value for a from its actual value was compared for each set of parameters. In every case, the destructively interfered distribution, corresponding to orthogonal pre- and post-selected states, gave the best fit even though in many cases, the detected intensity of the interfered distribution was many orders of magnitude smaller than the original, uninterfered distribution. This is due to the strong modulation in the signal shape, as demonstrated in Fig. 2c, which increases the ability to extract the magnitude of the small effect.

4. Conclusion

We have realized the first measurement of a weak value using an optical experiment. The effect, which depends on the interference between the eigenstates of unresolved eigenvalues, yielded a precise measurement of the very small birefringence induced separation of a laser beam. We used a computer simulation to demonstrate that this procedure may be useful for the detection of weak effects.

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